

# A MAGNETIC CURRENT FORMULATION FOR MODELLING DISCRETE COMPONENTS USING THE FDTD METHOD

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## ABSTRACT

A new formulation is proposed to simulate discrete components using the FDTD method. It uses the equivalence principle to replace the volume occupied by a component with its equivalent circuit. The actual size of the component can so be taken into account without performing any microscopic modelling. The parasitic elements traditionally associated with lumped representations are also suppressed.

## I. INTRODUCTION

The global electromagnetic analysis of loaded planar circuits appears as the only rigorous way to predict complex electromagnetic phenomena such as coupling and radiation occurring in passive and active, linear and non linear, microwave circuits. The FDTD method has been demonstrated to be particularly well suited to study these problems. As a 3D method, it provides a convenient way of modelling complex structures. As a time domain method, it provides a natural representation for non linear phenomena. Two main approaches have been proposed to connect discrete elements across FDTD cells. The first one is analogous to the microscopic approach as it involves a precise description of the different materials constitutive of the component [1]. Although this approach is theoretically more rigorous, it can only be applied to elementary components with simple descriptions. The second method is a lumped approach that uses an electrical equivalent circuit to represent the discrete element [2,3]. Unfortunately, the actual size and precise location of the component can not be taken into account easily and cumbersome distributing procedures are required [4]. Moreover, the parasitic capacitance associated with the cells supporting the lumped element has been demonstrated to spoil the results [5]. This paper proposes an alternative formulation that compromises between the two previous approaches. It consists in replacing all the volume

occupied by the component by its electrical equivalent circuit. As a consequence, it takes into account the actual size of the component and suppresses the parasitic effect of the loaded cells. However, it remains versatile and general since it relies on equivalent circuit approach. Basically, this approach uses the equivalence principle to represent the discrete component by a fictitious magnetic current density. This magnetic current surrounds the volume occupied by the component and simulates the voltage across its terminals. As a result, it produces the same fields outside the volume and zero fields inside it, which permits to vanish the effect of the inner FDTD cells. In this paper, this last property is illustrated and applied to improve the modelling of matched load terminating a shielded microstrip line.

## II. THEORY

Figure 1 represents a standard Yee cell loaded by a resistive voltage source. Using the principle of equivalence, an external equivalent problem can be obtained by assuming a perfect conductor inside volume abcdefgh and by adding an horizontal magnetic current  $M_s$  on the vertical part of its surface, as shown in the figure:

$$\mathbf{M}_s = -\mathbf{n} \times \mathbf{E} \quad (1)$$

In this equation, the voltage across the resistive source generates the electric field which is responsible for the surface magnetic current:

$$\mathbf{E} = \frac{V}{\Delta z} \mathbf{e}_z \quad (2)$$

In order to account for this magnetic current, the general form of Maxwell-Faraday curl equation has to be used for time-stepping the magnetic field:

$$\nabla \times \mathbf{E} = -\mathbf{M}_s - \mu \frac{\partial}{\partial t} \mathbf{H} \quad (3)$$

Using central difference in combination with (1) and (2) to update  $H_{x,i,j,k}$  leads to:

$$\begin{aligned} & \frac{E_z|_{i,j+1,k}^{n+1/2} - E_z|_{i,j,k}^{n+1/2}}{\Delta y} - \frac{E_y|_{i,j,k+1}^{n+1/2} - E_y|_{i,j,k}^{n+1/2}}{\Delta z} \\ &= \frac{V^{n+1/2}}{\Delta y \Delta z} - \mu \frac{H_x|_{i,j,k}^{n+1} - H_x|_{i,j,k}^n}{\Delta t} \end{aligned} \quad (4)$$

The voltage  $V$  is given by Ohm's law applied to the generator,

$$V = -V_s + R_s I_L \quad (5)$$

where current  $I_L$  can be expressed as the circulation of the magnetic field around the path ABCD

$$\begin{aligned} I_L^{n+1/2} = & \left( H_x|_{i,j-1,k}^n - \frac{H_x|_{i,j,k}^{n+1} + H_x|_{i,j,k}^n}{2} \right) \Delta x \\ & + \left( H_y|_{i,j,k}^n - H_y|_{i-1,j,k}^n \right) \Delta y \end{aligned} \quad (6)$$

Classically, in this equation, the updated component of the magnetic field has been time averaged but not the other components.

Putting (5) and (6) in (4) yields:

$$\begin{aligned} H_x|_{i,j,k}^{n+1} = & \frac{\frac{\mu}{\Delta t} - \frac{R_s}{2} \frac{\Delta x}{\Delta y \Delta z}}{\frac{\mu}{\Delta t} + \frac{R_s}{2} \frac{\Delta x}{\Delta y \Delta z}} H_x|_{i,j,k}^n + \frac{\frac{R_s}{\Delta y \Delta z}}{\frac{\mu}{\Delta t} + \frac{R_s}{2} \frac{\Delta x}{\Delta y \Delta z}} \left( H_x|_{i,j-1,k}^n \Delta x + H_y|_{i,j,k}^n \Delta y - H_y|_{i-1,j,k}^n \Delta y \right) \\ & - \frac{1}{\frac{\mu}{\Delta t} + \frac{R_s}{2} \frac{\Delta x}{\Delta y \Delta z}} \left( \frac{E_z|_{i,j+1,k}^{n+1/2} - E_z|_{i,j,k}^{n+1/2}}{\Delta y} - \frac{E_y|_{i,j,k+1}^{n+1/2} - E_y|_{i,j,k}^{n+1/2}}{\Delta z} \right) - \frac{\frac{V_s^{n+1/2}}{\Delta y \Delta z}}{\frac{\mu}{\Delta t} + \frac{R_s}{2} \frac{\Delta x}{\Delta y \Delta z}} \end{aligned} \quad (7)$$

Similar equations can be obtained to update  $H_{x,i,j-1,k}$ ,  $H_{y,i,j,k}$  and  $H_{y,i-1,j,k}$ . In addition to these equations,  $E_z|_{i,j,k}$  has to be forced to zero in order to respect the perfect conductor condition inside volume abcdefgh:

$$E_z|_{i,j,k} = 0 \quad (8)$$

This formulation can be generalized to deal with a generator extending over  $N$  cells along  $x$ . The global electric current flowing through the  $N$  loaded cell is calculated as the circulation of the magnetic field along the path as shown in figure 2. In addition, the  $H_y$  components located between

two loaded cells have to be forced to zero as they now lie inside the perfect conductor. The main advantage of this treatment is that it implies no assumption concerning the horizontal distribution of the vertical current.

### III RESULTS

As an illustration, a shielded microstrip line excited by a matched generator at its first extremity and terminated by a matched load at the other one is studied (figure 3). In this example, the discrete matched loads are used in order to avoid complex absorbing boundary conditions. The meshing involves 35 cells in the  $x$  direction, 100 cells in the

y direction and 25 cells in the z direction. The time step is 0.11 ps and 4545 iterations are performed. The width of the line extends over 5 cells which requires a distribution of the discrete elements along x. A first simulation (figure 4a) was performed using the classical lumped element formulation. The obtained return loss is equal to that produced by a matched load with a 15 fF shunt capacitance (i.e. the terminal capacitance of the open-ended line). A second simulation (figure 4b) was achieved using the magnetic current formulation. The return loss is equivalent for low frequencies but much better for higher frequencies (7 dB lower at 20 GHz). In this case, using equation (8), no displacement current can flow through the loaded cells, which means the corresponding capacitances vanish. As a result, the total parasitic capacitance at the end of the line decreases. The evaluation of the remaining capacitance gives 10 fF. This simple example confirms that the parasitic capacitance associated with the loaded cells has been suppressed.

#### IV. CONCLUSION

A new formulation has been presented to simulate discrete components using the FDTD method. It permits to represent the actual size of the component without performing its microscopic modelling. It has also been demonstrated to suppress the parasitic effects associated with lumped formulations. As a first application, this approach has been applied to simulate improved matched terminations in shielded structures, where no absorbing boundary conditions are necessary.

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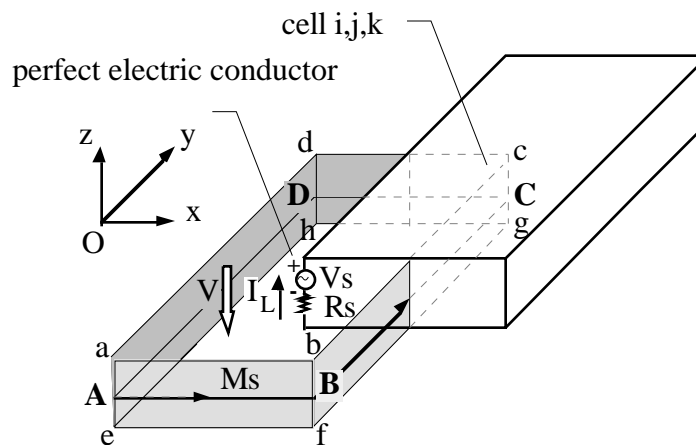


Figure 1 : General treatment of a discrete generator

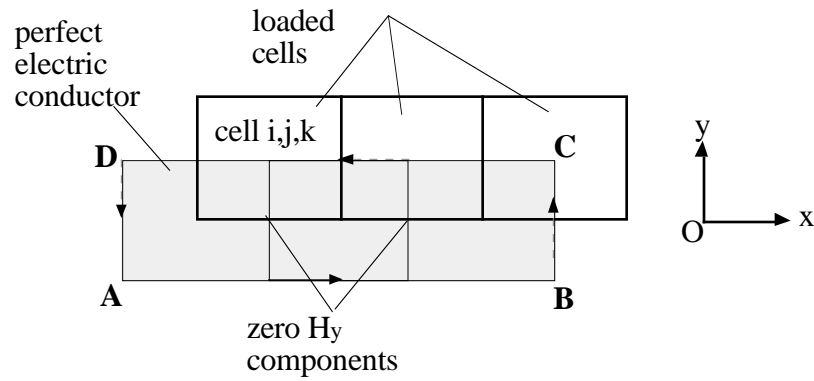


Figure 2: 3-cells loading configuration

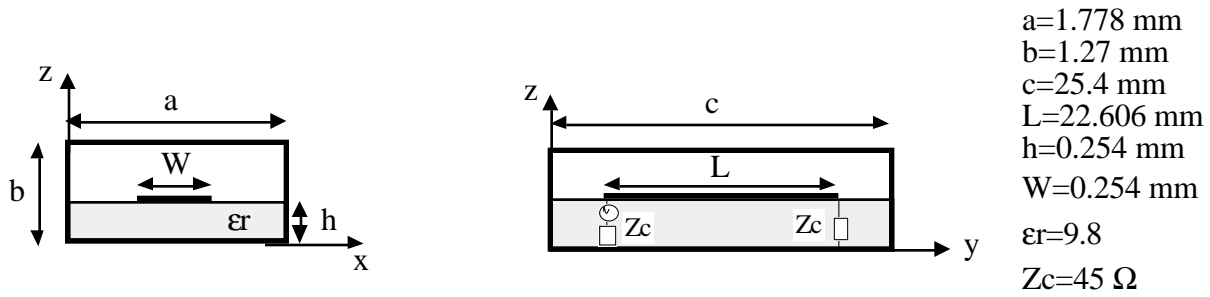


Figure 3: studied structure

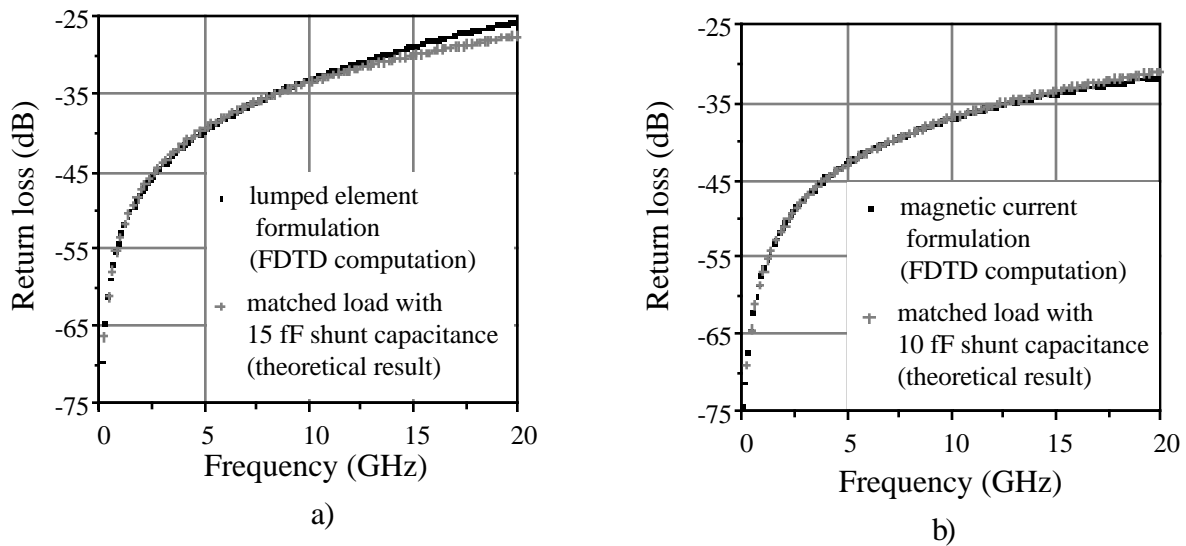


Figure 4: Computed return loss compared with that produced by a matched load with a shunt capacitance